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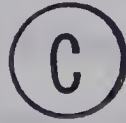
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GROWING HELICAL DENSITY WAVE IN INTRINSIC
SEMICONDUCTOR PLASMA

by



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A THESIS

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The undersigned certify that they have read, and recommend to the Faculty of Graduate Studies for acceptance, a thesis entitled Growing Helical Density Wave in Intrinsic Semiconductor Plasma, submitted by Kong Chong Ng, in partial fulfilment of the requirements for the degree of Master of Science.

ABSTRACT

The behaviour of a surface density wave situated in parallel electric and magnetic fields in a homogeneous plasma of a cylindrical intrinsic semiconductor bar is studied. The possibility of finite plasma densities at the surface of the semiconductor is included. A particularly simple model is used, requiring only a thermal equilibrium electron-hole plasma and low recombination surface. Explicit solutions of the basic equations are presented. The dynamic behaviour of the plasma is investigated by perturbation theory. The critical field at which instability appears and the frequency of the helix which thereby occurs are determined. Helical waves arise when the field exceeds a critical value. In this calculation, linear terms in the induced azimuthal magnetic field are retained. Two extreme cases of reasonably low and sufficiently high applied electric field are considered.

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NOTATION

R	radius of semiconductor sample.
n	electron-hole density.
\bar{n}	density at which no net recombinations or generations take place.
n_s	density at the boundary at which surface recombinations do not take place.
ξ	rate of creation or recombination of electron-hole pairs.
e	absolute value of electronic charge.
$\vec{\Gamma}$	particle flux.
\vec{E}	electric field.
\vec{B}	magnetic field.
A, Φ	parameters associated with azimuthal magnetic field.
T_{\pm}	temperatures of holes and electrons.
μ_{\pm}	mobilities of holes and electrons.
D_{\pm}	coefficient of diffusion of holes and electrons.
s	surface recombination velocity.
v_z	$= v_z^+ - v_z^-$ the sum of the absolute values of the longitudinal velocity, v_z is positive for positive electric field and is proportional to the current.
n_a	average electron-hole density.
I_0	modified zeroth order Bessel function.
J_1	first order Bessel function.
β_0, β_1	first and second zeros of I_0 .
K_0	first zero of J_1 .

c velocity of light.
 c_s velocity of sound in semiconductor.
 ω $= \omega_1 + i\omega_2$ frequency of the helical wave.
 m wave number of helical wave in azimuthal direction.
 k wave number of helical wave in z direction.
 $"-"$ and $"+"$ refer to electrons and holes.

Chapter 1: Introduction

1.1 Description of the system

The system to be studied consists of an electron-hole plasma situated in parallel electric and magnetic fields. The plasma is contained in a uniform cylindrical intrinsic semiconductor with diameter much smaller than the length of the sample. The diameter of the sample is of the order of a millimeter and the length is of the order of a centimeter, so that the end effects may be neglected. The electric field is produced by electrodes at the end of the semiconductor bar and the magnetic field by an electromagnet which is rotatable in a horizontal plane to complete the alignment of sample and field or by solenoid of diameter barely larger than that of the sample with which used.

When a charged particle moves through a region where an electric field and a magnetic field are present, the particle is subject to two forces. The electric force is parallel to the field. The magnetic force is at right angles both to the velocity of the particle and the magnetic field strength. When the electric field and magnetic field are simultaneously present and parallel to each other, the two motions are combined and the final path will be a helix. Whether the helix is right-handed or left-handed depends on the sign of the charge.

In a plasma, the longitudinal electric field tends to separate the superimposed positive and negative screws axially, which is equivalent to a rotation of one screw relative to the other. The screws of opposite charges will attenuate each other. The resulting charge separation creates both an azimuthal and a radial electric field. In the linear approximation, these fields together with the longitudinal field act on the unperturbed distribution to produce a radial and azimuthal flow of particles. If the unperturbed distribution has a radial gradient, this flow can feed particles from the main distribution into the screw, thus producing a growth of the perturbation. When the influx of particles is sufficiently large to overcome the dissipative effects of diffusion and recombination, the helical wave will grow.

It is expected that for a given value of electric field there will be a threshold value of magnetic field above which growth is observed. The effect corresponding to the steady state solution is given in Chapter 3. As the applied longitudinal magnetic field is increased, a screw type state is observed. This helical state will be unstable for sufficiently large magnetic field. This helical state solution is given in Chapter 4.

The variables of this problem are the density of electrons and holes, the electric field and the magnetic field. The electric and magnetic fields are externally applied and

are treated as independent variables. The density, the internal electric field and the induced magnetic field are the dependent variables.

1.2 Historical background

The helical plasma density wave (instabilities) was first proposed by Kadomtsev and Nedospasov¹ as an explanation of a gas discharge in a longitudinal magnetic field for the anomalous diffusion in the positive column. Johnson and Jerde² have given this theory a rigorous mathematical foundation. Glicksman³ suggested that a slightly generalized form of Kadomtsev and Nedospasov theory could explain the oscillistor effect in semiconductors. In its simplest form the oscillistor is a semiconductor bar that exhibits thermal voltage and current oscillations, when an electron-hole plasma is created by injection or light generation or collision ionization and sufficiently large electric and magnetic fields are applied. Current oscillation has been observed by several authors^{4,5} when a semiconductor plasma is placed in electric and magnetic fields. The helical motion of the plasma has been experimentally confirmed by Okamoto et al⁶ and Misawa et al⁷. Glicksman treated only the case of an injected plasma in the absence of a thermal background. However, the background plasma may be of great importance in a theory of the helical instability in a semiconductor plasma, as has been pointed out by others^{6,7}.

Holter⁸ applied Johnson's method and developed a theory for helical instabilities in solid state plasma that cover all injection levels in intrinsic, n-type and p-type semiconductors as well as insulators by taking the background of thermal carriers into account. A quantitative mathematical description of the helical wave in the surface density mode was formulated by Hurwitz and McWhorter⁹. Recently, the case of an inhomogeneous injected plasma in a cylindrical Ge rod possessing low surface recombination has been discussed by Schulz¹⁰.

According to the type of wave investigated, the existing theories can be separated into two groups. The theory introduced by Glicksman transferred the theory originally developed by Kadomtsev and Nedospasov for a helical density wave in the positive column of a gas plasma to electron-hole plasma in a semiconductor. It is assumed that recombination is dominant at the surface through the boundary condition that the density at the surface is zero and only bulk generation is taken into account. Modification is extended by other⁸, investigates a so-called gradient density wave which needs a gradient in the steady state carrier distribution, usually caused by a high surface recombination of the crystal. The second theory is introduced by Hurwitz and McWhorter for a homogeneous plasma of a cylindrical crystal. It is assumed that electron-hole recombination and regeneration depend on the local density. If the local density is larger (smaller) than some value, net

recombination (regeneration) will take place. A low surface recombination velocity is taken into consideration through the boundary condition at the walls.

None of the theoretical works in this field has included the forces due to the internal magnetic field in the azimuthal direction. It is assumed that the applied longitudinal magnetic field is usually very large compared with the azimuthal field and the latter may be neglected. As a consequence of the omission of this force, we can expect that the results of the above authors would be modified for large electric current when the induced azimuthal field becomes comparable to the longitudinal field.

1.3 Outline of the method

In the present calculation, we treat the surface density wave by including the induced magnetic field. The induced magnetic field is assumed to be small and only linear terms are kept. The behaviour of the plasma is investigated by perturbation and the effects of the induced magnetic field are found in the calculation. We follow Hurwitz and McWhorter by considering recombination and regeneration and assuming a small surface recombination velocity. Hurwitz and McWhorter assumed that the boundary properties are not different from the bulk. This approximation is inappropriate, for the density at the surface may be different from that in the bulk. We will

consider that there is a difference in bulk and surface density. Thus, two boundary conditions are to be applied. The first condition relates the flux at the boundary with the surface recombination velocity and the second condition ensures that the total rate of generation of electron-hole pairs is equal to the total rate of recombination.

The mode we are considering involves only the thermal equilibrium electron-hole plasma. The mobilities of electrons and holes are considered to be determined by collision of free electrons and holes with the lattice vibrations alone and collisions between electrons and holes are neglected. When a drift velocity is much less than the velocity of sound in the medium, the temperatures of electrons and holes are nearly equal to the temperature of the lattice. The mobilities and temperatures of electrons and holes are approximately constant at a reasonably small electric field. As the electric field is increased, the electrons and holes may then be considered as becoming hot and out of thermal equilibrium with the lattice. A significant departure of mobilities and temperatures from their low field values are then expected. The mobilities will now vary inversely with the square root of the electric field and the temperatures vary proportionally to the electric field¹¹.

To solve the problem, we make the assumption of quasi-neutrality. The differences in the density fluctuations of the two types of charges will be much smaller than the fluctuations

themselves. We may neglect the existence of space charge without excluding the internal electric field, for a very small space charge may set up a large internal electric field. These fields are established in a direction such that they try to equalise the densities. In the present problem, we want to investigate the density fluctuations that are equal for both types of charges. We neglect the difference in the density of the charges (i.e. no space charge) and still assume that an internal electric field exists. This assumption is known as quasi-neutrality.

The equations of motion of the system are obtained from Euler's equation of hydrodynamics. We treat the plasma as two fluids and write an equation for each of the two fluids, electrons and holes. The other hydrodynamics equation which we must use is the continuity equation for a fluid. Using the equations of motion and the equations of continuity for holes and electrons, the dispersion relation governing the growth and propagation of the helical wave will be investigated.

We study the steady state and the time dependent solutions of the system. In the steady state, the density at every point in space remains constant in time. We assume that density fluctuation can be written in the form of a series of zeroth order Bessel functions and we only retain terms linear in self induced magnetic field. To study the time dependent solution, we assume that deviations in density and in electric field from the steady state values are small and we only

retain terms linear in these deviations. We assume a screw type solution for the time dependent part of the densities and use the perturbation approach to find the effect of self induced magnetic field on the densities. Conditions for the instabilities in the screw type motion are then investigated. The condition for stability is found by requiring the imaginary part of the frequency to be equal to or greater than zero. This equation contains the wave number of the helical wave, the critical longitudinal magnetic field, the critical longitudinal electric field and the induced azimuthal magnetic field as parameters.

Chapter 2: Basic Equations

We consider an infinite uniform cylindrical intrinsic semiconductor (i.e. total number of free electrons and holes are equal) of radius R so that the end effects may be neglected. We assume the electrons and holes behave as a non-degenerate gas. In this cylinder, n_0 electrons and p_0 holes are distributed uniformly in thermal equilibrium. Since the semiconductor is an intrinsic one, we may write $n_0 = p_0 = \bar{n}$ and the following condition must therefore be fulfilled,

$$\nabla n_0 = \nabla p_0 = \nabla \bar{n} = 0.$$

When an electric field is applied to the semiconductor, the thermal equilibrium is upset. Quite large changes of n and p may however be obtained by applying a voltage of appropriate sign to the electrodes. A quasi-neutral density perturbation is created under the influence of an applied electric field. Electrons and holes are equal in number only in intrinsic semiconductors, while after excitation, the electron-hole pairs produced last only for a characteristic time called the life time. Let the disturbed electron concentration n be equal to $n_0 + n_1$, and the disturbed hole concentration p be equal to $p_0 + p_1$. Since we assume $n_0 = p_0 = \bar{n}$, unless $n_1 = p_1$ a space charge ρ will be set up giving for homogeneous material

$$\rho = e(p_1 - n_1).$$

We assume that n_1 must be nearly equal to p_1 unless a very strong electric field is present. This can only occur near the surface or in a region in which the impurity concentration varies very rapidly, giving rise to a high concentration gradient. We have from Poisson's equation

$$\text{div } \vec{E} = \frac{\rho}{\epsilon} = 4\pi e(p_1 - n_1) \frac{1}{\epsilon} ,$$

where ϵ is the permittivity.

For the present, we assume the generated plasma created by the applied electric field is quasi-neutral. Under the assumption that the times and distances of interest are much larger than the mean free times and paths of the free carriers, we consider the behaviour of a semiconductor in which a deviation from the equilibrium carrier concentration has been created. Clearly, if the influence causing the deviation is removed, the concentrations will return after a time to their equilibrium values.

Let the rate of creation or recombination of the electron-hole pairs be ξ . We then expressed the rate of change of n and p by mean of the equation^{1 2}

$$\left. \frac{d(n-\bar{n})}{dt} \right|_{\text{Recom.}} = -(n - \bar{n})\xi ,$$

where n is the electron-hole density, and \bar{n} is the density at which no net recombinations or generations take place. The term $-(n - \bar{n})\xi$ takes account of creation and recombination of electron-hole pairs. We assume no trapping of electrons and holes so that electrons and holes are generated in pairs.

The continuity equations for electrons and holes are

$$\frac{\partial(n-\bar{n})}{\partial t} + \nabla \cdot \vec{\Gamma}_{\pm} = -(n-\bar{n})\xi. \quad (1)$$

The subscript "-" and "+" denote the electrons and holes respectively. $\vec{\Gamma}_{\pm} = n\vec{v}_{\pm}$ are the particle flux vectors and \vec{v}_{\pm} is the average velocity of either component of the plasma.

The equations of motion are

$$\vec{\Gamma}_{\pm} + D_{\pm} \nabla n \mp \mu_{\pm} n \vec{E} \mp \mu_{\pm} \vec{\Gamma}_{\pm} \times \vec{B} = 0, \quad (2)$$

where we have used the quasi-neutral assumption and assumed that lattice scattering is dominant and neglected the collisions between electrons and holes. \vec{E} and \vec{B} are the electric and magnetic fields. μ_{\pm} are the mobilities and D_{\pm} are the diffusion coefficients of electrons and holes. μ_{\pm} and D_{\pm} are related by $D_{\pm} = (KT_{\pm}/e)\mu_{\pm}$, which is known as Einstein's relationship. T_{\pm} are the temperatures of electrons and holes.

Solve equations (2) for $\vec{\Gamma}_{\pm}$ explicitly and substitute in equations (1), the continuity equations are then written

$$\begin{aligned} \nabla \cdot [& \mu_{\pm}^2 D_{\pm}' \vec{B} (\nabla n \cdot \vec{B}) \mp \mu_{\pm}^2 \mu_{\pm}' n \vec{B} (\vec{B} \cdot \vec{E}) + D_{\pm}' \nabla n \\ & \mp D_{\pm}' \mu_{\pm} (\vec{B} \times \nabla n) \mp \mu_{\pm}' n \vec{E} + \mu_{\pm} \mu_{\pm}' n (\vec{B} \times \vec{E})] \\ & = \frac{\partial n}{\partial t} + (n-\bar{n})\xi, \end{aligned} \quad (3)$$

where

$$\mu_{\pm}' = \frac{\mu_{\pm}}{1 + \mu_{\pm}^2 B^2} \quad \text{and} \quad D_{\pm}' = \frac{D_{\pm}}{1 + \mu_{\pm}^2 B^2}.$$

We have used cylindrical coordinates for reasons of symmetry in solving $\vec{\Gamma}_{\pm}$. The z axis is taken along the direction of applied electric and magnetic fields. We have neglected $[B_{\theta}(R)/B_0]_z^2$ in comparison with unity. Here $B_{\theta}(R)$ is the induced azimuthal magnetic field at the boundary R. B_{θ} is simply related to the electric current through

$$B_{\theta}(r) = \frac{e}{c^2 r} \int_0^R n(r') v_z r' dr',$$

where $\vec{v}_z = \vec{v}_z^+ - \vec{v}_z^-$ and \vec{v}_z^{\pm} are the z components of the drift velocities and assumed to be independent of r and z. In the literature $B_{\theta}(r)$ has usually been neglected. We assume a simple form of $B_{\theta}(r)$

$$B_{\theta}(r) = \frac{e v_z n_a r}{c^2}, \quad (4)$$

where n_a is the average density of charges. The above form $B_{\theta}(r)$ seems reasonable if v_z is assumed independent of position.

Chapter 3: Steady State Solution

In this case, there are no variations in time and in space. The plasma will occupy the available volume with a distribution in density which depends on the sources and sinks for the plasma. The sink of the plasma will be primarily at the surface, with some volume recombination also allowable, so that the plasma density will have a maximum at the center of the cylinder and fall off to some smaller value at the surface. We assume that in steady state $E_\theta = 0$, $\frac{\partial n}{\partial \theta} = 0$, $\frac{\partial n}{\partial z} = 0$ and $\frac{\partial E_{0z}}{\partial z} = 0$. Where cylindrical coordinates are chosen with the axis along E_{0z} .

In the steady state the following condition must be satisfied.

$$\frac{\partial n}{\partial t} = 0.$$

(a) We neglect the induced azimuthal magnetic field and write the r components of equations (3), we obtain

$$-(n-\bar{n})\xi + D_{\pm}' \left(\frac{\partial^2 n}{\partial r^2} + \frac{1}{r} \frac{\partial n}{\partial r} \right) \mp \mu_{\pm}' \frac{1}{r} \frac{\partial}{\partial r} (rnE_{0r}) = 0. \quad (5)$$

By eliminating E_{0r} between the two equations of (5), the resulting equation is

$$\frac{\partial^2 n}{\partial r^2} + \frac{1}{r} \frac{\partial n}{\partial r} - \beta_0^2 n + \beta_0^2 \bar{n} = 0, \quad (6)$$

where

$$\beta_0^2 = \frac{\mu_+' + \mu_-'}{\mu_+'D_- + \mu_-D_+} \xi.$$

Solution of (6) is of the form

$$n = N_0 I_0(\beta_0 r) + \bar{n}, \quad (7)$$

where N_0 is the deviation of n at $r = 0$.

To complete the solution, the boundary conditions

$$\vec{\Gamma}_{\pm}(R) = s [n(R) - n_s], \quad (8)$$

$$\int_0^{2\pi} \int_0^R (n - \bar{n}) \xi r dr d\theta + 2\pi R s [n(R) - n_s] = 0 \quad (9)$$

are applied. Where $\vec{\Gamma}_{\pm}(R)$ are respectively the radial flux components of electrons and holes at the surface, s is the surface recombination velocity and $n(R) - n_s$ is the excess carrier density at the surface.

\bar{n} and n_s are connected by the first boundary condition through

$$-N_0 \left(D_{\pm}' + \mu_{\pm}' \frac{D_{+}' - D_{-}'}{\mu_{+}' + \mu_{-}'} \right) \frac{dI_0(\beta_0 r)}{dr} \Big|_{r=R} = N_0 s I_0(\beta_0 R) + s(\bar{n} - n_s) \quad (10)$$

and N_0 is given by second boundary condition

$$N_0 = \frac{-R \frac{s}{\xi} (\bar{n} - n_s)}{\int_0^R I_0(\beta_0 r) r dr + R \frac{s}{\xi} I_0(\beta_0 R)} \quad (11)$$

The radial component of the internal electric field E_{0r} may be determined from the condition that in the steady state, the radial flux of both the electron and hole must be equal.

$$E_{0r} = \frac{D_{+}' - D_{-}'}{\mu_{+}' + \mu_{-}'} \frac{1}{n} \frac{dn}{dr}. \quad (12)$$

(b) Including the induced azimuthal magnetic field, the r components of equations (13) are

$$-(n-\bar{n})\xi + D_{\pm}'\left(\frac{\partial^2 n}{\partial r^2} + \frac{1}{r}\frac{\partial n}{\partial r}\right) \mp \mu_{\pm}'\frac{1}{r}\frac{\partial}{\partial r}(rnE_{0r}) \\ + \mu_{\pm}\mu_{\pm}'\frac{1}{r}\frac{\partial}{\partial r}(rnE_{0z}B_{\theta}) = 0. \quad (13)$$

Eliminate E_{0r} between the two equations of (13), the equation is

$$\frac{\partial^2 n}{\partial r^2} + \left(\frac{1}{r} + Ar\right)\frac{\partial n}{\partial r} - (\beta_0^2 - 2A)n + \beta_0^2\bar{n} = 0, \quad (14)$$

where

$$A = \frac{\mu_{-}'\mu_{+}'(\mu_{-} + \mu_{+})}{\mu_{+}'D_{-}' + \mu_{-}'D_{+}'} \Phi E_{0z}, \\ \Phi = \frac{B_{\Phi}}{R}.$$

B_{Φ} is the induced magnetic field at the boundary $r = R$. Since n depends only on r , we may replace the partial derivatives by the total derivatives. Introducing

$$N = n - \frac{\beta_0^2}{\beta_0^2 - 2A} \bar{n},$$

the differential equation is then written as

$$\frac{d^2 N}{dr^2} + \left(\frac{1}{r} + Ar\right)\frac{dN}{dr} - (\beta_0^2 - 2A)N = 0. \quad (15)$$

If we put $A = 0$, which corresponds to zero induced magnetic field, equation (15) will then reduce to equation (6).

Now consider equation (15) using method of perturbation, the next approximation can be found. We may assume

the solution of (15) as a series of zeroth order Bessel function. The solution is written as

$$N_0[I_0(\beta_0 r) + a_1 I_0(\beta_1 r)],$$

where β_0 and β_1 are determined by the boundary conditions.

Substitute this expression in (15), multiply by $I_0(\beta_1 r) r dr$ and integrate over r from zero to R . Neglecting the higher order terms in Φ and using the orthogonal property of Bessel functions, we obtain an expression for a_1 whose linear term in A is

$$a_1 = \frac{A \int_0^R r^2 \frac{dI_0(\beta_0 r)}{dr} I_0(\beta_1 r) dr}{(\beta_1^2 - \beta_0^2) \int_0^R I_0^2(\beta_1 r) r dr}. \quad (16)$$

Hence approximation to the solution of equation (12) may be written as

$$n = N_0[I_0(\beta_0 r) + a_1 I_0(\beta_1 r)] + \frac{\beta_0^2}{\beta_0^2 - 2A} \bar{n}. \quad (17)$$

Using the value of β_0 , together with the boundary condition (8), β_1 can be determined from the following relation

$$\begin{aligned} & -\{(D_{\pm}' + \alpha \mu_{\pm}') N_0 \frac{\partial}{\partial r} [I_0(\beta_0 r) + a_1 I_0(\beta_1 r)]\}_{r=R} \\ & + \frac{\mu_{-}' \mu_{+}' (\mu_{-}' + \mu_{+}')}{\mu_{-}' + \mu_{+}'} \Phi R E_{0Z} [N_0 \{I_0(\beta_0 R) + a_1 I_0(\beta_1 R)\} \\ & + \frac{\beta_0^2}{\beta_0^2 - 2A} \bar{n} - n_s] \} \\ & = s N_0 [I_0(\beta_0 R) + a_1 I_0(\beta_1 R)] + s \left[\frac{\beta_0^2}{\beta_0^2 - 2A} \bar{n} - n_s \right], \end{aligned} \quad (18)$$

where

$$\alpha = \frac{D'_+ - D'_-}{\mu'_+ + \mu'_-}.$$

N_0 is determined by the second boundary condition (9) and is given as follows

$$N_0 = \frac{-\left(\frac{\beta_0^2}{\beta_0^2 - 2A} - 1\right)\bar{n} \frac{R^2}{2} - R \frac{S}{\xi} \left(\frac{\beta_0^2}{\beta_0^2 - 2A} \bar{n} - n_s\right)}{\int_0^R I_0(\beta_0 r) r dr + a_1 \int_0^R I_0(\beta_1 r) r dr + R \frac{S}{\xi} [I_0(\beta_0 R) + a_1 I_0(\beta_1 R)]}. \quad (19)$$

The radial component of the internal electric field is obtained from the condition of steady state.

$$E_{0r} = \frac{\alpha}{n} \frac{dn}{dr} - ar\Phi E_{0z}, \quad (20)$$

where

$$a = \frac{\mu_- \mu'_- - \mu_+ \mu'_+}{\mu'_- + \mu'_+}.$$

Chapter 4: Helical Solution

It is expected that for a given electric field there should be a critical magnetic field and a critical frequency at which growth is first observed. At this point, which we shall call threshold, the frequency ω and propagation constant k are both real, corresponding to neither growth nor attenuation. For longitudinal magnetic field below the critical field, the helical mode decays. Above the critical field the helical state is unstable. The imaginary part of the frequency depends critically on the longitudinal magnetic field. In this calculation, we shall include the induced azimuthal magnetic field up to the lowest order.

We begin with the same basic equations; the equations of motion

$$\vec{\Gamma}_{\pm} + D_{\pm} \nabla n + \mu_{\pm} n \vec{E} + \mu_{\pm} \vec{\Gamma}_{\pm} \times \vec{B} = 0 \quad (21)$$

and the equations of continuity

$$\frac{\partial(n-\bar{n})}{\partial t} + \nabla \cdot \vec{\Gamma}_{\pm} = -(n-\bar{n})\xi. \quad (22)$$

From equations (21), $\vec{\Gamma}_{\pm}$ are obtained explicitly, and $\vec{\Gamma}_{\pm}$ may be substituted into the continuity equations. To obtain $\vec{\Gamma}_{\pm}$, we write equations (21) explicitly as three equations in the cylindrical components of $\vec{\Gamma}_{\pm}$, and solve for the components of $\vec{\Gamma}_{\pm}$ respectively. From the expressions for these components, we write a vectorial expression for $\vec{\Gamma}_{\pm}$. This expression may

be written as

$$\vec{\Gamma}_{\pm} = - \left[\mu_{\pm}^2 D'_{\pm} \vec{B} (\nabla n \cdot \vec{B}) \mp \mu_{\pm}^2 \mu'_{\pm} n \vec{B} (\vec{B} \cdot \vec{E}) + D'_{\pm} \nabla n \right. \\ \left. \mp D'_{\pm} \mu_{\pm} (\vec{B} \times \nabla n) \mp \mu'_{\pm} n \vec{E} + \mu_{\pm} \mu'_{\pm} n (\vec{B} \times \vec{E}) \right] \quad (23)$$

Substituting expression (23) in equations (22), we obtain

$$\nabla \cdot \left[\mu_{\pm}^2 D'_{\pm} \vec{B} (\nabla n \cdot \vec{B}) \mp \mu_{\pm}^2 \mu'_{\pm} n \vec{B} (\vec{B} \cdot \vec{E}) + D'_{\pm} \nabla n \right. \\ \left. \mp D'_{\pm} \mu_{\pm} (\vec{B} \times \nabla n) \mp \mu'_{\pm} n \vec{E} + \mu_{\pm} \mu'_{\pm} n (\vec{B} \times \vec{E}) \right] \\ = \frac{\partial n}{\partial t} + (n - \bar{n}) \xi \quad (24)$$

Since we assume the induced azimuthal field is normally much smaller than the applied longitudinal field, we may replace $1 + \mu_{\pm}^2 B^2$ by $1 + \mu_{\pm}^2 B_0^2$, and write

$$\mu'_{\pm} = \frac{\mu_{\pm}}{1 + \mu_{\pm}^2 B_0^2} \quad \text{and} \quad D'_{\pm} = \frac{D_{\pm}}{1 + \mu_{\pm}^2 B_0^2}.$$

We obtain from (24)

$$- \frac{\partial n}{\partial t} + (\bar{n} - n) \xi + D'_{\pm} \nabla^2 n \mp \mu'_{\pm} \nabla \cdot (n \vec{E}) + \mu_{\pm} \mu'_{\pm} \nabla \cdot (n \vec{B} \times \vec{E}) \\ + \mu_{\pm}^2 D'_{\pm} \vec{B} \cdot \nabla (\vec{B} \cdot \nabla n) \mp \mu_{\pm}^2 \mu'_{\pm} \vec{B} \cdot \nabla (n \vec{B} \cdot \vec{E}) \mp \mu_{\pm} D'_{\pm} \nabla \cdot (\vec{B} \times \nabla n) \\ = 0. \quad (25)$$

To treat this problem, we suppose a fluctuation in the distribution of carriers to have originated at a certain moment of time, and introduce the perturbation terms n_1 and \vec{E}_1 defined by

$$n = n_0 + n_1, \\ \vec{E} = \vec{E}_0 + \vec{E}_1. \quad (26)$$

Where n_0 and \vec{E}_0 are the steady state values of n and \vec{E} , and n_1 and \vec{E}_1 are their time dependent parts, and are assumed small. Replacing n and \vec{E} with the above expressions in (25), and from which, subtracting the steady state equation, we obtain to first order of perturbations of n_1 and \vec{E}_1

$$\begin{aligned} & -n_1 \xi - \frac{\partial n_1}{\partial t} + D_{\pm} \nabla^2 n_1 \mp \mu_{\pm} \nabla \cdot (n_0 \vec{E}_1 + n_1 \vec{E}_0) \\ & + \mu_{\pm} \mu_{\pm}' \nabla \cdot [\vec{B} \times (n_0 \vec{E}_1 + n_1 \vec{E}_0)] + \mu_{\pm}^2 D_{\pm}' \vec{B} \cdot \nabla (\vec{B} \cdot \nabla n_1) \\ & \mp \mu_{\pm}^2 \mu_{\pm}' \vec{B} \cdot \nabla [\vec{B} \cdot (n_0 \vec{E}_1 + n_1 \vec{E}_0)] \mp \mu_{\pm} D_{\pm}' \nabla (\vec{B} \times \nabla n_1) = 0. \end{aligned} \quad (27)$$

To obtain the normal mode for the problem, we now assume a helical form for the density n_1 and the electrostatic potential V_1 from which the field \vec{E}_1 is derived. The general forms in cylindrical coordinates (r, θ, z) are

$$n_1 = [C + f(r)] \exp[i(\omega t + m\theta + kz)] \quad (28)$$

$$\text{and} \quad V_1 = g(r) \exp[i(\omega t + m\theta + kz)],$$

where ω is the frequency of the perturbation, m is the wave number in the azimuthal direction, k is the wave number along the z axis, and C can be determined by the boundary condition (9) at the walls.

$$C = \frac{-2 \int_0^R f(r) r dr - 2R \frac{S}{\xi} f(R)}{R^2 + 2R \frac{S}{\xi}}. \quad (29)$$

The radial part of n_1 takes the form $f(r) = f_0 J_m(K_0 r)$, where $J_m(K_0 r)$ is the Bessel function of the first kind of

order m , and f_0 is an arbitrary constant. The electric field \vec{E}_1 may be expressed in terms of the potential V according to the following set of equations

$$\begin{aligned}
 E_{1r} &= - \frac{\partial g}{\partial r} e^x, & \frac{1}{r} \frac{\partial E_{1r}}{\partial \theta} &= - \frac{im}{r} \frac{g}{r} e^x, \\
 \frac{\partial E_{1r}}{\partial r} &= - \frac{\partial^2 g}{\partial r^2} e^x, & \frac{\partial E_{1r}}{\partial z} &= - ik \frac{\partial g}{\partial r} e^x, \\
 E_{1\theta} &= - im \frac{V_1}{r}, & \frac{1}{r} \frac{\partial E_{1\theta}}{\partial \theta} &= m^2 \frac{g}{r^2} e^x, \\
 \frac{\partial(rE_{1\theta})}{\partial r} &= -im \frac{\partial g}{\partial r} e^x, & \frac{\partial E_{1\theta}}{\partial z} &= km \frac{g}{r} e^x, \\
 E_{1z} &= - ik V_1, & \frac{\partial E_{1z}}{\partial \theta} &= km g e^x, \\
 \frac{\partial E_{1z}}{\partial r} &= - ik \frac{\partial g}{\partial r} e^x, & \frac{\partial E_{1z}}{\partial z} &= k^2 g e^x,
 \end{aligned} \tag{30}$$

where $e^x = \exp[i(\omega t + m\theta + kz)]$.

Replace B_θ by its lowest order expression, that is the term linear in r

$$B_\theta \approx \frac{n_a v_z e r}{c^2},$$

and define

$$\Phi = \frac{B_\theta}{r} \approx \frac{n_a v_z e}{c^2}.$$

We obtain from (27) by keeping only linear terms in B_{0z} and Φ ,

$$\begin{aligned}
& D_{\pm}' \frac{1}{r} \frac{d}{dr} \left[r \frac{d(C+f(r))}{dr} \right] \mp \mu_{\pm}' \frac{1}{r} \frac{d}{dr} [r(C+f(r))E_{0r}] + [-\xi - i\omega \\
& - D_{\pm}' \frac{m^2}{r^2} - D_{\pm} k^2 \mp i\mu_{\pm} k E_{0z} + i\mu_{\pm} \mu_{\pm}' m \frac{B_{0z}}{r} E_{0r}] [C+f(r)] \\
& + \mu_{\pm} \mu_{\pm}' r E_{0z} \frac{d(C+f(r))}{dr} \Phi + [\mp i\mu_{\pm}^2 \mu_{\pm}' m B_{0z} E_{0z} \\
& + 2\mu_{\pm}' (\mu_{\pm} E_{0z} \pm i k D_{\pm}) - i k \mu_{\pm} \mu_{\pm}' r E_{0r}] [C+f(r)] \Phi \\
& = \mp \mu_{\pm}' \frac{1}{r} \frac{d}{dr} (r n_0 \frac{dg}{dr}) + [\pm \mu_{\pm}' n_0 \frac{m^2}{r^2} \pm \mu_{\pm} n_0 k^2 - i\mu_{\pm} \mu_{\pm}' m \frac{B_{0z}}{r} \frac{dn_0}{dr}] g(r) \\
& + \mu_{\pm} \mu_{\pm}' [2i n_0 k + i k r \frac{dn_0}{dr}] g(r) \Phi , \tag{31}
\end{aligned}$$

where E_{0r} has been derived previously in equation (20). Since all variables depend only on r , we have replaced the partial derivatives by total derivatives.

We substitute E_{0r} in equations (31) and define a new variable $\ell(r)$, such that

$$g(r) = \frac{1}{n_0(r)} \ell(r). \tag{32}$$

Rewriting equations (31) in terms of $f(r)$ and $\ell(r)$, we obtain

$$\begin{aligned}
& D_{\pm}' \frac{1}{r} \left[r \frac{d(C+f(r))}{dr} \right] \mp \mu_{\pm}' \frac{1}{r} \frac{d}{dr} \left[\alpha \frac{r}{n_0} \frac{dn_0}{dr} (C+f(r)) \right] \\
& + [-\xi - i\omega - D_{\pm}' \frac{m^2}{r^2} - D_{\pm} k^2 \mp i k \mu_{\pm} E_{0z} \\
& + i \alpha \mu_{\pm} \mu_{\pm}' m \frac{B_{0z}}{r} \frac{1}{n_0} \frac{dn_0}{dr}] [C+f(r)] \\
& + \mu_{\pm} \mu_{\pm}' r E_{0z} (1 \pm \frac{a}{\mu_{\pm}}) \frac{d(C+f(r))}{dr} \Phi \\
& + [\mp i \mu_{\pm}^2 \mu_{\pm}' m B_{0z} E_{0z} (1 \pm \frac{a}{\mu_{\pm}}) + 2\mu_{\pm} \mu_{\pm}' E_{0z} (1 \pm \frac{a}{\mu_{\pm}})
\end{aligned}$$

$$\begin{aligned}
& \pm 2ik\mu_{\pm}'D_{\pm} - ik\alpha\mu_{\pm}\mu_{\pm}' \frac{r}{n_0} \frac{dn_0}{dr}] [C+f(r)] \Phi \\
& = \mp \mu_{\pm}' \frac{1}{r} \frac{d}{dr} \left(r \frac{d\ell}{dr} \right) \pm \mu_{\pm}' \frac{1}{r} \frac{d}{dr} \left(\frac{r}{n_0} \frac{dn_0}{dr} \right) \ell(r) \\
& + [\pm \mu_{\pm}' \frac{m^2}{r^2} \pm \mu_{\pm} k^2 - i\mu_{\pm}^2 \mu_{\pm}' m \frac{B_0 z}{r} \frac{1}{n_0} \frac{dn_0}{dr}] \ell(r) \\
& + \mu_{\pm} \mu_{\pm}' [2ik + ik \frac{r}{n_0} \frac{dn_0}{dr}] \ell(r) \Phi . \tag{33}
\end{aligned}$$

Since $g(R)$ which is essentially the amplitude of the potential of the electric field at the boundary, must be finite and since $n(R)$ has finite values, due to the boundary condition at the surface, $\ell(R)$ must then be finite.

We may write the solution of (33) as Bessel function of order one

$$\begin{aligned}
f(r) &= f_0 J_1(K_0 r) \\
\ell(r) &= \ell_0 J_1(K_0 r), \tag{34}
\end{aligned}$$

where f_0 and ℓ_0 are independent of r . We substitute (34) in equations (33), multiply by $J_1(K_0 r) r dr$ and integrate over r from zero to R .

Define

$$P = \frac{1}{n_0} \frac{dn_0}{dr} = P_1 + P_2 A R^2, \tag{35}$$

where

$$\begin{aligned}
P_1 &= - \frac{1}{n_0} R \frac{S}{\xi} (\bar{n} - n_s) \frac{dI_0(\beta_0 r)}{dr} \frac{1}{D}, \\
P_2 &= - \frac{1}{n_0} \left[\bar{n} \frac{2}{\beta_0^2 R^2} \left(\frac{R^2}{2} + R \frac{S}{\xi} \right) \frac{dI_0(\beta_0 r)}{dr} \frac{1}{D} \right. \\
&\quad \left. + R \frac{S}{\xi} (\bar{n} - n_s) a_2 \frac{dI_0(\beta_0 r)}{dr} \frac{1}{D} \right],
\end{aligned}$$

and

$$a_2 = \frac{a_1}{A} ,$$

$$D = \left[1 + \frac{1}{4} \beta_0^2 R^2 \left(\frac{1}{2} + \frac{S}{R\xi} \right) \right] \left[\frac{R^2}{2} + R \frac{S}{\xi} \right] .$$

We obtain two algebraic equations whose coefficients are functions of the parameters describing the plasma and the wave parameters ω and k only.

$$\begin{aligned} & \left[D_{\pm}' \{M_3 - m^2(M_4 - M_5)\} \mp \alpha \mu_{\pm}' \{ (M_6 - M_8) + (M_7 - M_9) d R^2 \Phi \} \right. \\ & + (-\xi - i\omega - D_{\pm} k^2 \mp i k \mu_{\pm} E_{0Z}) (M_1 - M_2) R^2 \\ & + i \alpha \mu_{\pm} \mu_{\pm}' B_{0Z} m (M_{10} - M_{11}) \left. \right] f_0 + \left[\{ \mp i m \mu_{\pm}^2 \mu_{\pm}' B_{0Z} E_{0Z} (1 \pm \frac{a}{\mu_{\pm}}) \right. \\ & + 2 \mu_{\pm} \mu_{\pm}' (1 \pm \frac{a}{\mu_{\pm}}) E_{0Z} \mp 2 i k \mu_{\pm}' D_{\pm} \} (M_1 - M_2) R^2 \\ & + \mu_{\pm} \mu_{\pm}' E_{0Z} (1 \pm \frac{a}{\mu_{\pm}}) M_{12} R^2 - i \alpha k \mu_{\pm} \mu_{\pm}' (M_{13} - M_{14}) R^2 \left. \right] f_0 \Phi \\ & = \left[\mp \mu_{\pm}' (M_3 - m^2 M_4) \pm \mu_{\pm}' (M_6 + M_7 d R^2 \Phi) \mp \mu_{\pm} k^2 M_1 R^2 \right. \\ & \left. - i m \mu_{\pm} \mu_{\pm}' B_{0Z} M_{10} \right] \ell_0 + \mu_{\pm} \mu_{\pm}' \left[2 i k M_1 R^2 + i k M_{13} R^2 \right] \ell_0 \Phi . \end{aligned} \quad (36)$$

In equation (36)

$$d = \frac{\mu_- + \mu_+}{\frac{K}{e}(T_+ + T_-)} E_{0Z}$$

and all the M 's are expressed as integrals of Bessel function,

$$\begin{aligned} M_1 R^2 &= \int_0^R J_1^2(K_0 r) r dr , \\ M_2 R^2 &= \int_0^R C_1 J_1(K_0 r) r dr , \end{aligned}$$

$$\begin{aligned}
M_3 &= \int_0^R \frac{d}{dr} \left[r \frac{dJ_1(K_0 r)}{dr} \right] J_1(K_0 r) dr , \\
M_4 &= \int_0^R \frac{1}{r} J_1^2(K_0 r) dr , \\
M_5 &= \int_0^R \frac{C_1}{r} J_1(K_0 r) dr , \\
M_6 &= \int_0^R \frac{d}{dr} [r P_1 J_1(K_0 r)] J_1(K_0 r) dr , \\
M_7 &= \int_0^R \frac{d}{dr} [r P_2 J_1(K_0 r)] J_1(K_0 r) dr , \\
M_8 &= \int_0^R \frac{d}{dr} [r C_1 P_1] J_1(K_0 r) dr , \\
M_9 &= \int_0^R \frac{d}{dr} [r C_1 P_2] J_1(K_0 r) dr , \\
M_{10} &= \int_0^R P_1 J_1^2(K_0 r) dr , \\
M_{11} &= \int_0^R C_1 P_1 J_1(K_0 r) dr , \\
M_{12} R^2 &= \int_0^R r^2 \frac{dJ_1(K_0 r)}{dr} J_1(K_0 r) dr , \\
M_{13} R^2 &= \int_0^R r^2 P_1 J_1^2(K_0 r) dr , \\
M_{14} R^2 &= \int_0^R r^2 C_1 P_1 J_1(K_0 r) dr ,
\end{aligned} \tag{37}$$

where $C = -C_1 f_0$.

Dividing equations (36) by $\mu_{\pm}'(M_1 - M_2)$ and defining

$$\begin{aligned}
 N_1 &= \frac{M_3 - m^2(M_4 - M_5)}{M_1 - M_2} , \\
 N_2 &= \frac{M_6 - M_8}{M_1 - M_2} , \\
 N_3 &= \frac{M_{10} - M_{11}}{M_1 - M_2} , \\
 N_4 &= \frac{M_{12}}{M_1 - M_2} , \\
 N_5 &= \frac{M_{13} - M_{14}}{M_1 - M_2} , \\
 N_6 &= \frac{M_2}{M_1 - M_2} , \\
 N_7 &= \frac{M_7 - M_9}{M_1 - M_2} , \\
 N_8 &= \frac{M_8}{M_1 - M_2} , \\
 N_9 &= \frac{M_9}{M_1 - M_2} , \\
 N_{10} &= -\frac{m^2 M_5}{M_1 - M_2} , \\
 N_{11} &= \frac{M_{11}}{M_1 - M_2} , \\
 N_{12} &= \frac{M_{13}}{M_1 - M_2} ,
 \end{aligned} \tag{38}$$

we obtain two equations in f_0 and ℓ_0 of the form

$$\begin{aligned}
 A_+ f_0 + B_+ \ell_0 &= 0 , \\
 A_- f_0 + B_- \ell_0 &= 0 ,
 \end{aligned} \tag{39}$$

where

$$\begin{aligned}
 A_{\pm} = & \frac{KT_{\pm}}{e} N_1 \mp \alpha N_2 - \frac{\xi}{\mu_{\pm}} R^2 - \frac{i\omega}{\mu_{\pm}} R^2 - \frac{\mu_{\pm}}{\mu_{\pm}} \frac{KT_{\pm}}{e} k^2 R^2 \\
 & \mp \frac{\mu_{\pm}}{\mu_{\pm}} ikE_0 Z R^2 + im\alpha\mu_{\pm} B_0 Z N_3 + [\mp i\mu_{\pm}^2 B_0 Z E_0 Z mR(1 \pm \frac{a}{\mu_{\pm}}) \\
 & + 2\mu_{\pm} E_0 Z R(1 \pm \frac{a}{\mu_{\pm}}) \mp 2ikR\mu_{\pm} \frac{KT_{\pm}}{e} + \mu_{\pm} E_0 Z R(1 \pm \frac{a}{\mu_{\pm}}) N_4 \\
 & - i\alpha kR\mu_{\pm} N_5 \mp \alpha N_7 R d] R \Phi, \quad (40) \\
 B_{\pm} = & \pm(N_1 + N_{10}) \mp (N_2 + N_8) \mp \frac{\mu_{\pm}}{\mu_{\pm}} k^2 R^2 (1 + N_6) \\
 & + im\mu_{\pm} B_0 Z (N_3 + N_{11}) + [-2i\mu_{\pm} kR(1 + N_6) \\
 & - i\mu_{\pm} kR(N_5 + N_{12}) \mp (N_7 + N_9) dR] R \Phi.
 \end{aligned}$$

The dispersion relation for ω is calculated by setting the determinant of the equations equal to zero. The stability of a wave is investigated by treating either wave number or the frequency as a complex quantity. In the present calculation, we assume that the wave number k is real but the frequency is complex, thus $\omega = \omega_1 + i\omega_2$.

To have a non-trivial solution for f_0 and l_0 , we set the determinant of the coefficients of equations (39) equal to zero.

$$\begin{vmatrix} A_+ & B_+ \\ A_- & B_- \end{vmatrix} = 0. \quad (41)$$

At threshold, the real and imaginary parts may be separated. We then obtain two equations of the form

$$\begin{aligned}
A_1 \omega_1 + B_1 \omega_2 + C_{11} &= 0, \\
A_2 \omega_1 + B_2 \omega_2 + C_{22} &= 0,
\end{aligned} \tag{42}$$

where

$$\begin{aligned}
A_1 = B_2 &= \left(\frac{\mu_-}{\mu_+} - \frac{\mu_+}{\mu_-} \right) \left[m(N_3 - N_{11})B_{0z} - 2kR(1 + N_6) R \Phi \right. \\
&\quad \left. - kR(N_5 + N_{12}) R \Phi \right], \\
A_2 = -B_1 &= - \left[-(N_1 + N_{10}) \left(\frac{1}{\mu_+} + \frac{1}{\mu_-} \right) + (N_2 + N_8) \left(\frac{1}{\mu_+} + \frac{1}{\mu_-} \right) \right. \\
&\quad \left. + \frac{\mu_- + \mu_+}{\mu_+ \mu_-} k^2 R^2 (1 + N_6) \right. \\
&\quad \left. + dR(N_7 + N_9) \left(\frac{1}{\mu_+} + \frac{1}{\mu_-} \right) R \Phi \right], \\
C_{11} &= \left[\frac{KT_+}{e} N_1 - \alpha N_2 - \frac{\xi R^2}{\mu_+} - \frac{\mu_+}{\mu_+} \frac{KT_+}{e} k^2 R^2 + 2\mu_+ E_{0z} R \left(1 + \frac{a}{\mu_+} \right) R \Phi \right. \\
&\quad \left. + \mu_+ E_{0z} R \left(1 + \frac{a}{\mu_+} \right) N_4 R \Phi - \alpha N_7 dR^2 \Phi \right] \left[-(N_1 + N_{10}) \right. \\
&\quad \left. + (N_2 + N_8) + \frac{\mu_-}{\mu_-} k^2 R^2 (1 + N_6) + d(N_7 + N_9) R^2 \Phi \right] \\
&\quad - \left[- \frac{\mu_+}{\mu_+} k E_{0z} R^2 + \alpha \mu_+ B_{0z} N_3 - 2\mu_+ k R \frac{KT_+}{e} R \Phi \right. \\
&\quad \left. - \alpha \mu_+ k R^2 N_5 \Phi \right] \left[m\mu_- B_{0z} (N_3 + N_{11}) \right. \\
&\quad \left. - 2\mu_- k R^2 (1 + N_6) \Phi - \mu_- k R^2 (N_5 + N_{12}) \Phi \right] \\
&\quad + \left[\frac{KT_-}{e} N_1 + \alpha N_2 - \frac{\xi R^2}{\mu_-} - \frac{\mu_-}{\mu_-} k^2 R^2 + 2\mu_- E_{0z} \left(1 - \frac{a}{\mu_-} \right) R \Phi \right. \\
&\quad \left. + \mu_- E_{0z} R^2 \left(1 - \frac{a}{\mu_-} \right) N_4 \Phi + \alpha N_7 dR^2 \Phi \right] \left[-(N_1 + N_{10}) \right. \\
&\quad \left. + (N_2 + N_8) + \frac{\mu_+}{\mu_+} k^2 R^2 (1 + N_6) + dR^2 (N_7 + N_9) \Phi \right]
\end{aligned}$$

$$\begin{aligned}
& + \left[\frac{\mu_-}{\mu_+} k E_0 Z R^2 + m \alpha \mu_- B_0 Z N_3 + 2 \mu_- k R^2 \frac{KT_-}{e} \Phi \right. \\
& \quad \left. - \alpha \mu_- k R^2 N_5 \Phi \right] \left[m \mu_+ B_0 Z (N_3 + N_{11}) \right. \\
& \quad \left. - 2 \mu_+ k R^2 (1 + N_6) \Phi - \mu_+ k R^2 (N_5 + N_{12}) \Phi \right], \\
C_{22} = & \left[\frac{KT_+}{e} N_1 - \alpha N_2 - \frac{\xi}{\mu_+} R^2 - \frac{\mu_+}{\mu_+} \frac{KT_+}{e} k^2 R^2 + 2 \mu_+ E_0 Z R^2 \left(1 + \frac{a}{\mu_+}\right) \right. \\
& + \mu_+ E_0 Z R \left(1 + \frac{a}{\mu_+}\right) N_4 R \Phi - \alpha N_7 d R^2 \Phi \left. \right] \left[m \mu_- B_0 Z (N_3 + N_{11}) \right. \\
& \left. - 2 \mu_- k R^2 (1 + N_6) \Phi - \mu_- k R^2 (N_5 + N_{12}) \Phi \right] \\
& + \left[- \frac{\mu_+}{\mu_+} k E_0 Z R^2 + m \alpha \mu_+ B_0 Z N_3 - 2 \mu_+ k R^2 \frac{KT_+}{e} \Phi \right. \\
& \quad \left. - \alpha \mu_+ k R^2 N_5 \Phi \right] \left[- (N_1 - N_{10}) + (N_2 + N_8) \right. \\
& \quad \left. + \frac{\mu_-}{\mu_-} k^2 R^2 (1 + N_6) + d R^2 (N_7 + N_9) \Phi \right] \\
& - \left[\frac{KT_-}{e} N_1 + \alpha N_2 - \frac{\xi}{\mu_-} R^2 - \frac{\mu_-}{\mu_-} \frac{KT_-}{e} k^2 R^2 + 2 \mu_- E_0 Z R^2 \left(1 - \frac{a}{\mu_-}\right) \right. \\
& \quad \left. + \mu_- E_0 Z R^2 \left(1 - \frac{a}{\mu_-}\right) N_4 \Phi - \alpha N_7 d R^2 \Phi \right] \\
& \left[m \mu_+ B_0 Z (N_3 + N_{11}) - 2 \mu_+ k R^2 (1 + N_6) - \mu_+ k R^2 (N_5 + N_{12}) \Phi \right] \\
& + \left[\frac{\mu_-}{\mu_+} k E_0 Z R^2 + m \alpha \mu_- B_0 Z N_3 + 2 \mu_- k R^2 \frac{2KT_-}{e} \Phi \right. \\
& \quad \left. - \alpha \mu_- k R^2 N_5 \Phi \right] \left[- (N_1 + N_{10}) + (N_2 + N_8) \right. \\
& \quad \left. + \frac{\mu_+}{\mu_+} k^2 R^2 (1 + N_6) + d R^2 (N_7 + N_9) \Phi \right].
\end{aligned}$$

In order to effect a considerable simplification in the mathematics, it is assumed from here onward that $\mu_{\pm}^2 B_{\theta}^2 < \mu_{\pm}^2 B_0^2$ $\ll 1$, we may replace $1 + \mu_{\pm}^2 B_0^2$ by one, and all the primes in (43) may be dropped.

From equations (42), the imaginary frequency may be written

$$\omega_2 = \frac{\begin{vmatrix} A_1 & -C_{11} \\ A_2 & -C_{22} \end{vmatrix}}{\begin{vmatrix} A_1 & B_1 \\ A_2 & B_2 \end{vmatrix}}. \quad (44)$$

The threshold for the onset of instability will occur when $\omega_2 = 0$, because the ω variation of the wave first changes from attenuation to growth at this point. Therefore, we set

$$A_1 C_{22} - A_2 C_{11} = 0. \quad (45)$$

The parameters of the system are of course the magnetic and electric fields applied to the plasma and the induced azimuthal magnetic field. Note that B_0 now represents the critical magnetic field.

We wish now to calculate both the extreme cases for weak and sufficiently strong electric fields. It is known that the electrons' and holes' temperatures and mobilities are functions of the applied electric field and expressed¹¹, for very weak applied electric field, as

$$\mu_{\pm}^0 E_0 \ll c_s,$$

$$\begin{aligned}
\mu_{\pm} &= \mu_{\pm}^0 \left[1 - \frac{1}{2} \left(\frac{\mu_{\pm}^0 E_{0z}}{c_s} \right)^2 \frac{3\pi}{32} \right], \\
T_{\pm} &= T_{\pm}^0 \left[1 + \left(\frac{\mu_{\pm}^0 E_{0z}}{c_s} \right)^2 \frac{3\pi}{32} \right].
\end{aligned}
\tag{46}$$

for sufficiently strong applied electric field, as

$$\begin{aligned}
\mu_{\pm}^0 E_{0z} &\gg c_s, \\
\mu_{\pm} &= \mu_{\pm}^0 \left(\frac{c_s}{\mu_{\pm}^0 E_{0z}} \right)^{\frac{1}{2}} \left(\frac{32}{3\pi} \right)^{\frac{1}{4}}, \\
T_{\pm} &= T_{\pm}^0 \frac{\mu_{\pm}^0 E_{0z}}{c_s} \left(\frac{3\pi}{32} \right)^{\frac{1}{2}} = T_{\pm}^0 \frac{\mu_{\pm}^2 E_{0z}^2}{c_s^2} \frac{3\pi}{32}.
\end{aligned}
\tag{47}$$

Where T_{\pm}^0 and μ_{\pm}^0 are temperatures and mobilities of electrons and holes at vanishing electric field, c_s is the velocity of sound in the medium and $T_{\pm} > T_{\pm}^0$. In weak electric field $T_{\pm} - T_{\pm}^0$ is proportional to E_{0z}^2 , thus we see that the effective temperatures of electrons and holes are raised slightly above the equilibrium value, but that this is a second order effect, so that the usual assumption that $T_{\pm} = T_0$ is justified. We see that $\mu_{\pm} - \mu_{\pm}^0$ are also proportional to E_{0z}^2 and that provided $\mu_{\pm}^0 E_{0z} \ll c_s$, it is a good approximation that μ_{\pm} is independent of E_{0z} and equal to μ_{\pm}^0 .

Substituting the values of A_1 , A_2 , C_{11} , C_{22} from (43) in (45) and after performing some algebraic manipulation, we obtain for both weak and strong applied electric field the equation for the requirement of stability.

$$\begin{aligned}
& b(\mu_- + \mu_+)B_0 Z \left[H_1 \frac{\Phi R}{B_0 Z} + mH_2 Z_1^{\frac{1}{2}} + H_3 Z_1 \frac{\Phi R}{B_0 Z} + mH_4 Z_1^{\frac{3}{2}} \right. \\
& \left. + H_5 Z_1^2 \frac{\Phi R}{B_0 Z} \right] + H_6 + H_7 Z_1 + H_8 Z_1^2 + H_9 Z_1^3 = 0 \quad (48)
\end{aligned}$$

where

$$\begin{aligned}
Z_1 &= \frac{k^2 R^2}{N_1} \\
b &= \frac{E_0 Z R}{N_1^{\frac{1}{2}} \frac{K}{e} (T_+ + T_-)} \\
H_1 &= G_1^2 G_4 / N_1^{\frac{1}{2}} - 2G_7 G_1 G N_1^{\frac{1}{2}}, \\
H_2 &= -G_3 G_1, \\
H_3 &= -[G_1(2G_4 G_6 - G_5)/N_1^{\frac{1}{2}} - 2G_7(G_1 + G_6 G)N_1^{\frac{1}{2}}], \\
H_4 &= G_3 G_6, \\
H_5 &= G_6(G_4 G_6 - G_5)/N_1^{\frac{1}{2}} - 2G_7 G_6 N_1^{\frac{1}{2}}, \\
H_6 &= G_1^2 G, \\
H_7 &= -G_1(2G_6 G - G_1), \\
H_8 &= G_6(G_6 G + 2G_1), \\
H_9 &= -G_6^2,
\end{aligned}$$

and

$$\begin{aligned}
G &= 1 - \beta_0^2 R^2 / N_1, \\
G_1 &= (N_1 + N_{10} - N_2 - N_8) / N_1, \\
G_2 &= N_2 / N_1, \\
G_3 &= (N_3 + N_{11}) / N_1, \\
G_4 &= 2 + N_4, \\
G_5 &= 2(1 + N_6) + (N_5 + N_{12}), \\
G_6 &= 1 + N_6, \\
G_7 &= (N_7 + N_9) / N_1,
\end{aligned} \quad (49)$$

$$G_8 = N_3/N_1 ,$$

$$G_9 = N_5 ,$$

$$\text{if } \mu_{\pm}^0 E_{0Z} \ll c_s ,$$

$$b(\mu_+ + \mu_-)B_{0Z} = \frac{E_{0Z} R}{2N_1^{1/2} \frac{K}{e} T_0} (\mu_-^0 + \mu_+^0) B_{0Z} ,$$

$$\text{if } \mu_{\pm}^0 E_{0Z} \gg c_s ,$$

$$b(\mu_+ + \mu_-)B_{0Z} = \frac{(\frac{32}{3\pi})^{3/4} c_s^{3/2} R}{N_1^{1/2} \frac{K}{e} T_0} \frac{\mu_-^{0^{1/2}} + \mu_+^{0^{1/2}}}{\mu_-^0 + \mu_+^0} \frac{1}{E_{0Z}^{1/2}} B_{0Z} .$$

Equation (48) gives the smallest value of E_{0Z} for which instability may occur. The general behaviour is as expected; larger magnetic fields are required to cause instabilities when lower electric fields are applied.

At the onset of instability, we also have the condition that the derivative of $\omega_2=0$ with respect to wave number k must be equal to zero. When this condition is satisfied, a large number of waves have nearly the same imaginary parts, their amplitudes decay or increase in time at the same rate. Thus producing an enhanced effect. Thus

$$\begin{aligned} & -b(\mu_- + \mu_+)B_{0Z} \left[mH_2 + 2H_3 Z_1^{1/2} \frac{\Phi R}{B_{0Z}} + 3mH_4 Z_1 + 4H_5 Z_1^{3/2} \frac{\Phi R}{B_{0Z}} \right] \\ & = 2H_7 Z_1^{1/2} + 4H_8 Z_1^{3/2} + 6H_9 Z_1^{5/2} . \end{aligned} \quad (50)$$

Equation (50) together with equation (48) give the value of $k = k_c$ in terms of the parameters E_{0Z} and B_{0Z} ; such that ω_2 is nearly zero for the neighbouring points of k_c .

Eliminating E_{0z} between (48) and (50), we obtain an equation relating the wave number in z direction k and the critical magnetic field B_{0z} and the induced magnetic field.

$$\begin{aligned}
& 2H_5H_9 \frac{\Phi R}{B_{0z}} Z_1^{9/2} + 3mH_4H_9 Z_1^4 + 4H_3H_9 \frac{\Phi R}{B_{0z}} Z_1^{7/2} \\
& + m(H_4H_8 + 5H_2H_9)Z_1^3 + (2H_3H_8 + 6H_1H_9 - 2H_5H_7) \frac{\Phi R}{B_{0z}} Z_1^{5/2} \\
& + m(3H_2H_8 - H_4H_7)Z_1^2 + 4(H_1H_8 - H_5H_6) \frac{\Phi R}{B_{0z}} Z_1^{3/2} \\
& + m(H_2H_7 - 3H_4H_6)Z_1 + 2(H_1H_7 - H_3H_6) \frac{\Phi R}{B_{0z}} Z_1^{1/2} \\
& - m H_2H_6 = 0.
\end{aligned} \tag{51}$$

Another quantity we wish to calculate as a function of the critical magnetic field is the frequency of rotation of the helix. We solve for ω_1 from equations (42), which may be written

$$\omega_1 = \frac{\begin{vmatrix} -C_{11} & B_1 \\ -C_{22} & B_2 \end{vmatrix}}{\begin{vmatrix} A_1 & B_1 \\ A_2 & B_2 \end{vmatrix}}, \tag{52}$$

which gives the real part of ω at the stability boundary. Where $A_1, A_2, B_1, B_2, C_{11}$ and C_{22} are given in (43) and noted that

$$B_1 = -A_2,$$

$$B_2 = A_1.$$

Therefore ω_1 may be written

$$\omega_1 R^2 = - \frac{A_1 C_{11} + A_2 C_{22}}{A_1^2 + A_2^2}. \tag{53}$$

Substituting the values from (43), after much algebra, we obtain an equation relating the real frequency ω_1 , the electric field (E_{0Z}) variable b and the wave number (k) variable Z_1 . Here again we consider the two extreme cases for weak and sufficiently strong electric field. The frequency ω_1 is given by if $\mu_{\pm}^0 E_{0Z} \ll c_s$,

$$\begin{aligned} & b_1 [H_{11} + H_{12}b(\mu_- + \mu_+)\Phi R + H_{13}Z_1 + H_{14}Z_1^2] \\ &= \frac{m}{2}(\mu_- - \mu_+)B_{0Z} (H_{15} + H_{16}Z_1 + H_{17}Z_1^2) \\ &+ \frac{1}{2}(\mu_- - \mu_+) \Phi R (H_{18}Z_1^{1/2} + H_{19}Z_1^{3/2} + H_{20}Z_1^{5/2}), \end{aligned} \quad (54)$$

if $\mu_{\pm}^0 E_{0Z} \gg c_s$,

$$\begin{aligned} & b_1 [H_{11} + H_{12}b(\mu_- + \mu_+)\Phi R + H_{13}Z_1 + H_{14}Z_1^2] \\ &= m(\mu_- - \mu_+)\left(1 + \frac{\mu_- \mu_+}{\mu_+^2 + \mu_-^2}\right)B_{0Z} (H_{15} + H_{16}Z_1 + H_{17}Z_1^2) \\ &+ (\mu_- - \mu_+)\left(1 + \frac{\mu_- \mu_+}{\mu_+^2 + \mu_-^2}\right)\Phi R (H_{18}Z_1^{1/2} + H_{19}Z_1^{3/2} + H_{20}Z_1^{5/2}), \end{aligned} \quad (55)$$

where

$$\begin{aligned} b_1 &= \frac{\mu_- + \mu_+}{\mu_- \mu_+} \frac{\omega_1 R^2}{N_1 \frac{K}{e}(T_+ + T_-)}, \\ H_{11} &= G_1^2, \\ H_{12} &= -2G_1 G_7 N_1^{1/2}, \\ H_{13} &= -2G_1 G_6, \\ H_{14} &= G_6^2, \\ H_{15} &= G_1 [G_3(1 - G_2) - G_1 G_8], \end{aligned} \quad (56)$$

$$H_{16} = -G_3 (G_1 + G_6 - G_2 G_6) + 2G_1 G_6 G_8 ,$$

$$H_{17} = G_6 (G_3 - G_6 G_8) ,$$

$$H_{18} = G_1 [G_1(2 + G_9) - G_5(1 - G_2)] / N_1^{\frac{1}{2}} ,$$

$$H_{19} = [G_5(G_1 + G_6 - G_2 G_6) - 2G_1 G_6(2 + G_9)] / N_1^{\frac{1}{2}} ,$$

$$H_{20} = G_6 [G_6(2 + G_9) - G_5] / N_1^{\frac{1}{2}} .$$

If $\mu_{\pm}^0 E_{0Z} \ll c_S$,

$$\mu_- - \mu_+ = \mu_-^0 - \mu_+^0 ,$$

$$b_1 = \frac{\mu_-^0 + \mu_+^0}{\mu_-^0 \mu_+^0} \frac{\omega_1 R^2}{2N_1 \frac{K}{e} T_0} ,$$

if $\mu_{\pm}^0 E_{0Z} \gg c_S$,

$$(\mu_- - \mu_+) \left(1 + \frac{\mu_- \mu_+}{\mu_+^2 + \mu_-^2}\right) = c_S^{\frac{1}{2}} \left(\frac{32}{3\pi}\right)^{\frac{1}{4}} (\mu_-^{0\frac{1}{2}} - \mu_+^{0\frac{1}{2}}) \left(1 + \frac{\mu_-^{0\frac{1}{2}} \mu_+^{0\frac{1}{2}}}{\mu_+^0 + \mu_-^0}\right) \frac{1}{E_{0Z}^{\frac{1}{2}}} ,$$

$$b_1 = c_S^{\frac{1}{2}} \left(\frac{32}{3\pi}\right)^{\frac{1}{4}} \frac{\mu_+^{0\frac{1}{2}} + \mu_-^{0\frac{1}{2}}}{\mu_-^{0\frac{1}{2}} \mu_+^{0\frac{1}{2}} (\mu_+^0 + \mu_-^0)} \frac{\omega_1 R^2}{N_1 \frac{K}{e} T_0} \frac{1}{E_{0Z}^{\frac{1}{2}}} .$$

Thus, the frequency can be calculated when k and E_{0Z} are determined.

Chapter 5: Discussion

Steady state solution (17) is of the standard form containing Bessel functions in the limit $\Phi = 0$. However the coefficient N_0 of I_0 vanishes if $\bar{n} = n_s$ and $\Phi = 0$. If $\Phi = 0$, the steady state solution gives an increase in density on the axis and a decrease at boundary. If the boundary properties are not different from those in the bulk, i.e. $\bar{n} = n_s$, and we neglect the effect of the self magnetic field, the steady state solution is $n = n_s$. Thus we have a constant density. All previous authors have applied only the first boundary condition, in their result, the coefficient of the Bessel function of density is arbitrary. By the second boundary condition we imposed, the coefficient of the Bessel function of density is not an arbitrary constant but depends on the density at the walls. We believe that the second condition is necessary, therefore $n_s \neq \bar{n}$.

Equations (48), (51), (54) and (55) are derived on the assumption that the use of the relations (46) and (47) and the assumption $\mu_{\pm}^2 B_0^2 / Z \ll 1$ is valid. The general form of equations (48) and (51) is the same for both the weak field case and the strong field case. The only difference is in the E_0 / Z dependence of μ_{\pm} and T_{\pm} on the left hand side of (48). For the frequency of rotation of the helical wave, equations (54) and (55) are obtained for weak and strong field cases

respectively. It is possible to compute the critical values of E_{0z} , k and ω_1 from equations (48), (51), (54) and (55) for the two extreme field cases.

For $m = 0$, in (48), the magnetic field does not enter the expression at all. A solution of the form (28) with $m = 0$ is a pure density wave with no screw motion. It can easily be proved from equation (48) that $m = 0$ mode will always decay. Since any perturbations will be opposed by diffusion, neglecting the Φ terms corresponds to assuming that the force due to the induced magnetic field of the current is smaller than the diffusion force which maintains the steady state. If we take Φ terms into account, there will be some values of $\Phi \geq \Phi_c$, the critical induced magnetic field, and it causes a driving force larger than the diffusion force, instabilities may occur.

The helical type instability which corresponds to $m = \pm 1$ are considered in the following. The threshold for the onset of instability will occur when (48) becomes an equality, i.e. $\omega_2 = 0$. The parameters of the system are of course the wave number and the electric and magnetic fields applied to the plasma. For any fixed value $Z_1^{1/2}(k)$ obtained from (51), there will be corresponding values of E_{0z} and B_{0z} which will represent the onset of growing of that wave number. The general behaviour is as expected, namely, larger magnetic fields are required to cause instabilities when lower electric

fields are applied.

For small values of B_{0z} , $Z_1(k)$ approaches a constant value, and frequency varies linearly with B_{0z} as well as linearly in Φ . If $\mu_- = \mu_+$, the frequency of the helix yield a minimum frequency of zero, there will be no helical mode. It is to be expected from symmetry that either direction of propagation of the helix is equally possible. For $\mu_- < \mu_+$, the frequency changes sign, which indicates a change in direction of the propagating waves.

The general form of equations (48), (51), (54) and (55) are similar to the corresponding equations of other authors^{9,10}, $B_{0z} \propto \frac{1}{\mu_- + \mu_+} \frac{1}{E_{0z}}$ and $\omega_1 \propto (\mu_- - \mu_+)B_{0z}$, except that our equations contain the influence of azimuthal magnetic field.

In order to get approximate solutions of the above quantities without the use of computers, we make the approximation that $\frac{s}{R\xi} \ll 1$ and expand the Bessel functions in terms of their arguments and keep terms up to third power of the arguments. By using the values $s = 30\text{cm/sec.}$, $\frac{1}{\xi} = 1400\mu\text{ sec}$ and $R = 0.058\text{cm.}$ from Hurwitz and McWhorter⁹, the approximation $\frac{s}{R\xi} \ll 1$ seems reasonable. Using the expansions of the Bessel function, approximate values of all the H's in (48), (51), (54) and (55) are evaluated. Threshold values of $E_{0z}B_{0z}$, wave number k and the frequency of the helix may then be obtained from the corresponding equation.

Approximate solutions for $b(E_0)_Z B_{0Z}$ and $Z_1^{\frac{1}{2}}(k)$ at the threshold take the same form for both weak and strong field cases. The threshold value for $Z_1^{\frac{1}{2}}(k)$ is obtained from equation (51), which gives

$$\begin{aligned}
 Z_1^{\frac{1}{2}}(k) = & \frac{2^{\frac{1}{2}}}{3} \left(\frac{3}{10} K_0^2 R^2 - \frac{9}{8} \beta_0^2 R^2 \frac{s}{R\xi} \frac{\bar{n}-n_s}{\bar{n}} \right)^{\frac{1}{2}} \\
 & + \frac{1}{m} \left[- \frac{144}{24^{\frac{1}{2}}} \frac{1 + \frac{11}{15} K_0^2 R^2}{\beta_0^2 R^2 \frac{s}{R\xi} \frac{\bar{n}-n_s}{\bar{n}}} \right. \\
 & \left. + \frac{64}{3x(24)^{\frac{1}{2}}} \frac{1}{\beta_0^2 R^2 \frac{s}{R\xi} \frac{\bar{n}-n_s}{\bar{n}}} \left(\frac{3}{10} K_0^2 R^2 - \frac{9}{8} \beta_0^2 R^2 \frac{s}{R\xi} \frac{\bar{n}-n_s}{\bar{n}} \right) \right] \frac{\Phi R}{B_{0Z}}.
 \end{aligned}
 \tag{57}$$

Using the value of $Z_1^{\frac{1}{2}}(k)$ in (57), we get also an approximate threshold value of $E_0 B_{0Z}$

$$\begin{aligned}
 b(\mu_- + \mu_+) B_{0Z} = & \frac{64x(2)^{\frac{1}{2}}}{4m\beta_0^2 R^2 \frac{s}{R\xi} \frac{\bar{n}-n_s}{\bar{n}} - 40x(48)^{\frac{1}{2}} \left(\frac{3}{10} K_0^2 R^2 - \frac{9}{8} \beta_0^2 R^2 \frac{s}{R\xi} \frac{\bar{n}-n_s}{\bar{n}} \right)^{\frac{1}{2}} \frac{\Phi R}{B_{0Z}}} \\
 & \times \left[\left(\frac{3}{10} K_0^2 R^2 - \frac{9}{8} \beta_0^2 R^2 \frac{s}{R\xi} \frac{\bar{n}-n_s}{\bar{n}} \right)^{\frac{1}{2}} \right. \\
 & \left. + \left\{ - \frac{432}{48^{\frac{1}{2}}} \frac{1 + \frac{11}{15} K_0^2 R^2}{\beta_0^2 R^2 \frac{s}{R\xi} \frac{\bar{n}-n_s}{\bar{n}}} \right. \right. \\
 & \left. \left. + \frac{8}{24^{\frac{1}{2}}} \frac{\frac{3}{10} K_0^2 R^2 - \frac{9}{8} \beta_0^2 R^2 \frac{s}{R\xi} \frac{\bar{n}-n_s}{\bar{n}}}{\beta_0^2 R^2 \frac{s}{R\xi} \frac{\bar{n}-n_s}{\bar{n}}} \right\} \frac{\Phi R}{B_{0Z}} \right]
 \end{aligned}
 \tag{58}$$

$Z_{1\frac{1}{2}}(k)$ increases as the surface concentration increases, and the threshold for instability increases to larger values of E_{0z} and B_{0z} .

The critical frequency ω_1 is obtained from (54) for weak electric field case

$$b_1 = \frac{1}{2}(\mu_- - \mu_+) \frac{C_n}{C_d}, \quad (59)$$

and from (55) for strong electric field case

$$b_1 = (\mu_- - \mu_+) \left(1 + \frac{\mu_- \mu_+}{\mu_-^2 + \mu_+^2}\right) \frac{C_n}{C_d}, \quad (60)$$

where

$$\begin{aligned} C_d = & 6 \left(\frac{3}{10} K_0^2 R^2 - \frac{9}{8} \beta_0^2 R^2 \frac{s}{R\xi} \frac{\bar{n} - n_s}{\bar{n}} \right) \\ & - \frac{4320}{12^{\frac{1}{2}}} \frac{\left(\frac{3}{10} K_0^2 R^2 - \frac{9}{8} \beta_0^2 R^2 \frac{s}{R\xi} \frac{\bar{n} - n_s}{\bar{n}} \right)^{\frac{1}{2}}}{\beta_0^2 R^2 \frac{s}{R\xi} \frac{\bar{n} - n_s}{\bar{n}}} \frac{\Phi R}{B_{0z}}, \\ C_n = & m \frac{9}{8} \beta_0^2 R^2 \frac{s}{R\xi} \frac{\bar{n} - n_s}{\bar{n}} B_{0z} - \frac{81}{12^{\frac{1}{2}}} \frac{m}{\beta_0^2 R^2 \frac{s}{R\xi} \frac{\bar{n} - n_s}{\bar{n}}} R \Phi \\ & + \frac{18}{12^{\frac{1}{2}}} \left(\frac{3}{10} K_0^2 R^2 - \frac{9}{8} \beta_0^2 R^2 \frac{s}{R\xi} \frac{\bar{n} - n_s}{\bar{n}} \right)^{\frac{1}{2}} R \Phi. \end{aligned} \quad (61)$$

Equation (57) gives just the magnitude of k . For a wave of the form $\exp[i(\omega t + m\theta + kz)]$, it is evident that only the relative signs of k , m and ω are of significant. We choose the signs of k and m to make $\omega > 0$ in (59) and (60)

and to satisfy (58). The resulting signs of k , m and ω for various directions of E_{0z} and B_{0z} are the same as those given in Table 1. of Hurwitz and McWhorter. The sense and motion of the helical wave may be deduced from the signs of k , m and ω .

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